Acoustic Gas Lenses for Light

It has been proposed [1] that sound waves in acoustic waveguides might form lens systems useful for guiding light beams over reasonable distances. The pressure variations of the acoustic waves cause variations in the index of refraction which may have lens-like properties. The analysis of light guiding by a traveling acoustic waveguide mode has been given [2]. Such a wave is an "alternating gradient" focusing system [3] with alternating converging and diverging lenses of approximately equal strength. Such a system has net focusing properties [4] since the average radius in the diverging lenses is less for such a system than in the converging lenses. Acoustic lenses without longitudinal variations have also been utilized for the modulation of lasers, but with solids instead of gases [5]. Although liquids have a higher density of scattering particles than gases, the higher compressibility of gases make the net effect in the latter comparable to that in liquids for at least some applications. It is thus desirable to demonstrate some of the experimental effects in acoustic gas lenses for several acoustic modes.

We report here the deflection and transformation of a He-Ne 6328 Å laser beam due to circularly symmetric modes of lower frequencies, one with longitudinal variation and another with two nodal planes along the axis of propagation. For the first mode, the alternate compression and rarefaction of the gas yield a system of alternate converging and diverging lenses in time. The second one corresponds to an iterative system of converging and diverging lenses, and has resultant converging properties as discussed for the traveling acoustic mode.

The circularly symmetric mode with one circular node of pressure, called [6] mode (0, 1), has a variation of pressure of the form [7]

$$p = p_M \cos\left(\frac{q\pi z}{l}\right) J_0\left(\frac{3.83}{a}r\right) e^{j\omega t} \quad (1)$$

where a is the radius, l is the length of the cavity, q is the number of planes of zero pressure along the axis of the cylinder, and ω is the angular frequency of the acoustic wave. An incremental change dN of the number of molecules of the gas per unit volume is proportional to the pressure: $dN = N_m p/\gamma RT$, where N_m is the Avogadro's number, γ is the ratio of specific heat, R is the gas constant, and T is the absolute temperature. The index of refraction n is related to the number N of molecules per unit volume by the Lorentz-Lorenz formula [8]: $(n^2-1)/(n^2+2) = AN/N_m$, where A is the molar refractivity of the gas. For the variation of pressure as in (1), we have a distribution of the index of refraction given by

$$n = n_0\left[1 + \frac{3A}{\gamma RT} p_M J_0(kr) \cdot \cos\left(\frac{q\pi z}{l}\right) e^{j\omega t}\right]. \quad (2)$$

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Modes with $q=0$ correspond to the cutoff frequency of the cavity, and there is no variation of n along the z axis. The transversal and radial variations are represented in Fig. 1 for a mode with axial symmetry and one radial period. The stationary wave with two nodal planes (Fig. 2) corresponds to $q=2$.

A block diagram of the experiment is shown in Fig. 3. The cylindrical cavity of dimensions $a=1$ cm, $l=7$ cm is filled with propane, C_3H_8, at 8.55 atmospheres pressure. The acoustic waves are excited by a University UXT 5 "tweeter" which can handle a peak power of 50 watts. In spite of the high quality factor of the cavity, the coupling between the loudspeaker and the resonator is very low and the maximum variation of pressure is evaluated at 0.37 percent of the static pressure. A pressure-monitoring transducer has been introduced in the end of the cavity as shown in Fig. 3. This transducer uses a single crystal of barium titanate. The square plate crystal of surface 1.91 mm² and thickness $t=0.31$ mm, has its sides along the [001] axis, and is used as an expander bar. The pressure is given by $p_M = e/tg_{33}$, where e is the variable electric potential between the two faces and g_{33} the piezoelectric constant equal to [9] 57.3×10^{-3} volt-meter per newton. The values $p_M = 6500$ dynes/cm² obtained for the mode without z variation, and $p_M = 2800$ dynes/cm² for the stationary mode, are only comparative because the transducer has not been calibrated, and gives only an order-of-magnitude of the amplitude of the pressure.

Near the optical axis, we can approximate the Bessel function of zero order by a quadratic function. The mode (0, 1) at the cutoff frequency presents a variation of index of refraction of the form

$$n \approx n_0\left[1 - \frac{2r^2}{b^2}\right] \quad (3)$$

where $b^2 = 8\gamma RT/3k^2Ap_M$ and $k = 3.83/a$. The frequency of this mode is given by $f = 3.83c/2\pi a$ and is equal to 14.2 kHz for a velocity of the sound $c = 245$ m/s. During the first half-period, p_M and b^2 are positive. We have the linear expression given by (44) of Kogelnik [10], between the parameters x_2 and x_2' of the output plane (Fig. 4) and the corresponding input quantities x_1 and x_1'. For an incoming beam parallel to the axis, $x_1'=0$ and after a distance l of the medium, the slope of the output beam is equal to $x_2' = -4lx_1/b^2$. This deflection corresponds to a converging effect. During the second half-period p_M and b^2 are negative. We have now an analogous relation with hyperbolic functions and the deflection of a parallel incoming beam is given by: $x_2' = 4lx_1/|b|^2$. We have a diverging effect of the same value as the converging one. Using the complex parameter defined by (72) of Kogelnik [10], we

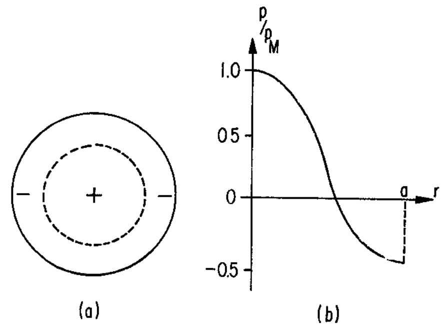

Fig. 1. Variation of the pressure for the mode (0, 1). (a) Transversal variation. (b) Radial variation.

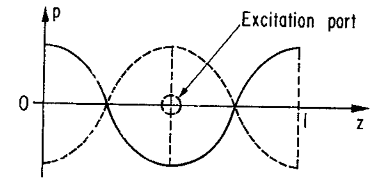

Fig. 2. Variation of the pressure along the axis of propagation for the mode (0, 1) with two nodal planes along the axis.

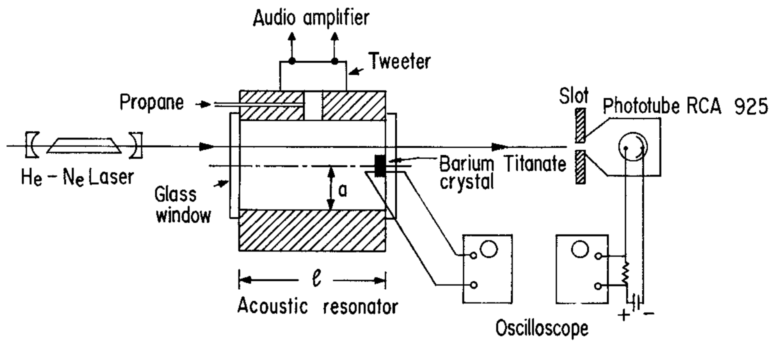

Fig. 3. Diagram of the experiment.

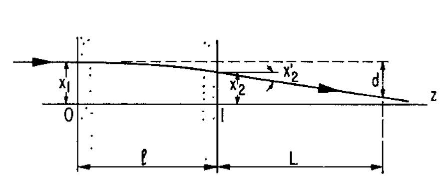

Fig. 4. Deflection due to the mode (0, 1) with no longitudinal variation.

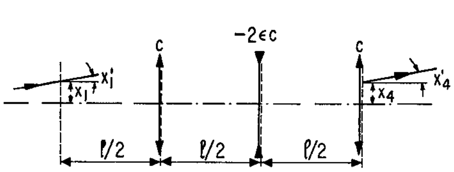

Fig. 5. Equivalent optical system to the mode (0, 1) with two nodal planes along the axis during the first half-period.

can analyze the transformation of the beam by the medium. During the first half-period, the size of the beam is given by (103) of Kogelnik [10], and we have another expression for the radius of curvature of the wavefront. In our experiment, w_0 given by (99) of Kogelnik [10], is equal to 0.65 mm when p is maximum. An incoming beam, with a spot size larger than w_0, is contracted by the medium, and we have a negative value for the radius of curvature of the wavefront. During the second half-period, we have an expression with hyperbolic functions, and the effect is always defocusing.

For the stationary wave with $q=2$, the equivalent optical system is represented (Fig. 5) by a thin diverging lens of convergence $(-2\epsilon c)$ situated between two converging lenses of convergence c. Input and output ray slopes are shown as x_1' and x_4', respectively. A half-wavelength of medium has a convergence [1]: $2c = 3n_0 Alk^2 p_M / 2\pi\gamma RT$. The factor ϵ, equal to or less than unity, is used to take into account an asymmetry between the compression and the rarefaction of the medium, and a nonlinear variation of the index of refraction with the pressure. The frequency of this mode is given by

$$f = \frac{c}{2\pi a}\left[\left(\frac{3.83}{a}\right)^2 + \left(\frac{2\pi}{l}\right)^2\right]^{1/2}. \qquad (4)$$

For our dimensions, this is equal to 14.7 kHz. The ray matrix of the system is given by the product of the matrices corresponding to each lens defined by (30) of Kogelnik [10]. The convergence of the system is equal to $c_1 = 2c(1-\epsilon) + lc^2 - l^2c^3/2$. During the second half-period, we have a converging lens of convergence $2\epsilon c$ situated between two diverging lenses of convergence $(-\epsilon c)$. The system has a converging effect: $c_2 = 2c(1-\epsilon) + lc^2 + l^2c^3/2$, which is almost the value obtained during the first half-period.

The value of the deflection can be obtained by measuring the displacement of a slot in front of the beam. The intensity of the light going through the slot is represented by equally spaced pulses. When we move the slot toward the edge of the deflected beam, the spacing between two pulses decreases, and they coincide when the slot is situated at the center of the maximum deflected beam. For the mode (0, 1) with no longitudinal variation, we have measured an angle of 3.3 minutes between the converging and the diverging deflections, for an incoming beam situated at 3 mm from the center of the cavity. A deflection of 23.4 minutes was obtained for the mode (2, 0) with no longitudinal variation which has two diametral planes of zero pressure. Near half the radius of the cavity, the Bessel function of zero order has a linear variation. Then the medium is equivalent to a prism, and for the modes with no longitudinal variation, the deflection is larger than near the center of the cavity.

For the mode (0, 1) with two nodal planes along the axis, the deflection is very small, and for a position of the slot near the inflection point of the Gaussian beam, we have an almost linear variation of the intensity with the deflection. By measuring the displacement of the slot, which gives the same variation, we can measure the deflection. We obtain an angle of 2.9 seconds for an incoming beam situated at 3 mm from the center of the cavity. This value is 100 times larger than the calculated value with $\epsilon = 1$ and corresponds to an asymmetry of 2.5 percent between the compression and the rarefaction. A measurement of the pressure along the axis and the diameter of the cavity would have been useful to determine whether the asymmetry actually exists or whether other perturbations explain the larger deflection than that expected.

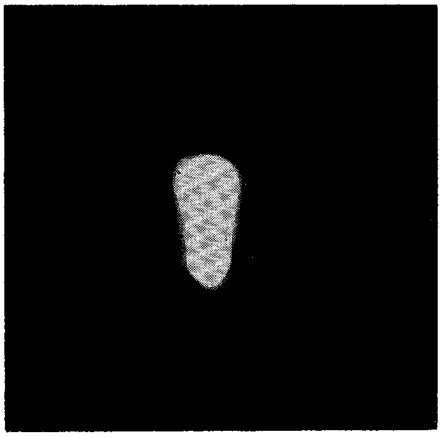

Fig. 6. Deflection due to the mode (0, 1) with no longitudinal variation.

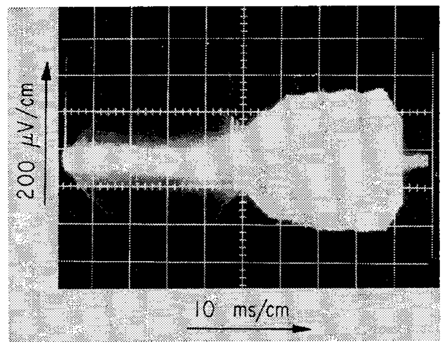

Fig. 7. Output of the phototube for the deflection of a beam near the center of the cavity, due to the mode (0, 1) with no longitudinal variation.

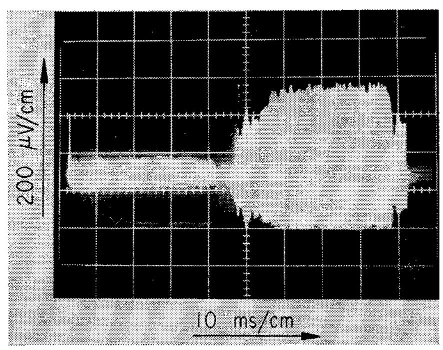

Fig. 8. Output of the phototube for the deflection of a beam near the center of the cavity, due to the mode (0, 1) with two nodal planes along the axis.

Figure 6 shows the deflection obtained from the (0, 1) mode with no longitudinal variation of pressure. This figure represents the exposure of many cycles of the acoustic wave in sweeping the beam back and forth on the recording film. Figure 7 is a different representation of the deflection from this mode, with the phototube response shown on the oscillograph utilizing the slot method of detecting small deflections as described previously. Figure 8 shows a representation similar to that of Fig. 7 but for the mode (0, 1) with two nodal planes along the axis. Note the asymmetry of this picture which shows the net converging effect.

The work reported shows the possibility of obtaining appreciable deflection and focusing with acoustic gas lenses, and the differences between modes. The net focusing effect of the alternating-gradient system is observed. Additional measurements should be made with calibrated transducers in various places in order to compare experimental values with the calculated ones, and to determine the important asymmetries of the alternating-gradient system. Comparisons of liquid and gas acoustic lenses would also be desirable.

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The Resonant Frequency of Interdigital Filter Elements

In a recent correspondence Nicholson [1] has described a method of predicting the center frequency of a bandpass interdigital filter. On the basis of existing techniques the design of this type of filter usually results in an error in the center and bandedge frequencies of the filter. The most important reason for this is the arbitrary manner in which the lengths of the fingers (i.e., center conductor) must be shortened at the open end [2]. By using

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